V24 - Stochastic Dynamics simulations of a photosynthetic vesicle

where bioinformatics meets biophysics

Introduction: prelude photosynthesis

II Process view and geometric model of a chromatophore vesicle Tihamér Geyer & V. Helms (Biophys. J. 2006a, 2006b)

III Stochastic dynamics simulations

T. Geyer, Florian Lauck & V. Helms (J. Biotechnol. 2007)

IV Parameter fit through evolutionary algorithm T. Geyer, X. Mol, S. Blaß & V. Helms (PLoS ONE 2010)



Content of final exam (March 7, 2016)

Lecture	Slides relevant for exam	Lecture	Slides relevant for exam
1	18-24	13	The main ideas
2	1-14, 18-20	14	1-18
3	17-26	15	1-22
4	All	16	1-11, 14
5	1-32	17	41-48
6	None	18	1-14
7	None	19	18-41
8	All	20	None
9	None	21	None
10	5-17	22	None
11	8-27	23	None
12	The main ideas	24	None



Relevant are also the assignments ! (theoretical parts, not the programming parts)

"Genomes To Life" Computing Roadmap (NIH/DOE)



Biological Complexity



Bacterial Photosynthesis 101





Photosynthesis – cycle view



2 cycles: electrons protons



LH1 / LH2 / RC — a la textbook

ATPase **Collecting photons** LH2: 8 $\alpha\beta$ dimers downhill transport Periplasm of excitons $LH2 \rightarrow LH1 \rightarrow RC$ LH1: 16 $\alpha\beta$ dimers RC (b) TH-ILLIAN THE BC THE THEIL THEIL LH-I DH-

B800, B850, Car.



Hu *et al*, 1998 6

The Cytochrome *bc*₁ complex



the "proton pump"





Berry, etal, 2004

X-ray structures known always forms a dimer



The F_oF₁-ATP synthase I

at the end of the chain: producing ATP from the H+ gradient





The F₁F₀-ATP synthase

"...mushroom like structures observed in AFM images..."

➔ ATPase is "visible" 1 ATPase per vesicle



limited throughput of the ATPase



Gräber et al, 1991, 1999

The electron carriers

Cytochrome c: carries electrons from bc_1 to RC

- heme in a hydrophilic protein shell
- 3.3 nm diameter, water-soluble



Ubiquinone UQ10:

carries electron–proton pairs from RC to bc_1

long (2.4 nm)
 hydrophobic
 isoprenoid tail,
 membrane-soluble



taken from Stryer







Chromatophore vesicle: typical form in *Rh. sphaeroides*



Lipid vesicles 30–60 nm diameter H⁺ and cyt *c* inside

average chromatophore surface vesicle, 45 nm Ø: 6300 nm²

Vesicles are really small!



Photon capture rate of LHC's

relative absorption spectrum of LH1/RC and LH2

sun's spectrum at ground (total: 1 kW/m²)



typical growth condition: 18 W/m² Feniouk et al, 2002





LH1 / LH2 / RC — native

electron micrograph and density map



Siebert et al, 2004 125 * 195 Ų, γ = 106°

	Area per:	per vesicle (45 nm)
LH1 monomer (hexagonal)	146 nm²	
LH1 dimer	234 nm²	
LH2 monomer	37 nm²	
LH1 ₂ + 6 LH2	456 nm²	11

Chromatophore surface vesicle, 45 nm Ø: 6300 nm²



Photon processing rate at the RC

Which process limits the RCs turnover?



1 RC can serve 1 LH1 + 3 LH2 = 44 γ/s → 25 ms Milano *et al.* 2003

+ binding, charge transfer
≈ 50 ms per quinol (estimate)

with 2e- H+ pairs per quinol

→ 40–50 γ/s per RC $\approx 22 \text{ QH}_2/\text{s}$

 $LH1_2 + 6 LH2 \triangleq 456 \text{ nm}^2 \rightarrow 11 LH1 \text{ dimers including } 22 \text{ RCs}$ on one vesicle

480 Q/s can be loaded @ 18 W/m² per vesicle

Modelling of internal processes at reaction center





All individual reactions with their individual rates k together determine the overall conversion rate R_{RC} of a single RC.

Thick arrows : flow of the energy from the excitons through the cyclic charge state changes of the special pair Bchl (P) of the RC.

Rounded rectangles : reservoirs



*bc*¹ Placement — Diffusional limits?

Roundtrip times maximal capacity of the carriers:

 $T = T_{RC} + T_{bc1} + T_{Diff}$

Cytochrome c₂:

T_{Diff}≈3μs $T_{pc} \approx 1 \text{ ms}$ $T_{bc1} \approx 12 \text{ ms}$ $T_{round-trip} = 13 \text{ ms} \rightarrow \leq 3 \text{ cyt c per vesicle}$ sufficient to carry e-'s available: 22 cyt c per vesicle

Quinol:

 $T_{_{RC}} \approx 50 \text{ ms}$ $T_{_{bc1}} \approx 23 \text{ ms}$ $T_{_{Diff}} \approx 1 \text{ ms}$

 $T_{round-trip} = 75 \text{ ms} \rightarrow \leq 7 \text{ Q per vesicle}$ sufficient to carry e-'s.

available: 100 Q per vesicle





Diffusion is not limiting

 \rightarrow poses no constraints on the position of bc_1

Parameters

protein	throughput	H+	total number	rate determined from	explained
	per protein	equivalents	per avg.		in section
	(natural units)	per protein	vesicle of		
		[1/s]	45 nm diameter		
LH2	$10 \ \gamma/s$	20	60	absorption spectra +	III A
LH1 dimer	$2 \times 14 \gamma/s$	56	10	+ light intensity of 18 W/m ²	III A
RC	22 QH2/s	88	20	QH2 (un)binding	III B
bc1 dimer	$\leq 2 imes 42$ c2/s	168	3 (10)	measured activity at $\Delta pH = 0$	III C
ATPase	≤ 100 ATP/s	400	1	measured throughput	III D
cytochrome c_2	80 e ⁻ /s	160	20	(un)binding at the bc1	V A (III B, III C)
ubiquinone	$10 \times 2(e^-H^+)/s$	40	100	(un)binding at the RC and the bc1	VA(IIIB,IIIC)



reconstituted LH1 dimers in planar lipid membranes explain intrinsic curvature of vesicles

Drawn after AFM images of Scheuring *et al* of LH1 dimers reconstituted into planar lipid membranes.







Values fit nicely to the proposed arrangement of LH1 dimers, when one assumes that they are stiff enough to retain the bending angle of 26° that they would have on a spherical vesicle of 45 nm diameter and taking into account the length of a single LH1 dimer of about 19.5 nm.

s fit nicely to the proposed arrangen

Proposed setup of a chromatophore vesicle

yellow arrows: diffusion of the protons out of the vesicle via the ATPase and to the RCs and bc1s. At the "poles" green/red: the ATPase light blue: the bc1 complexes

Increased proton density close to the ATPase suggests close proximity of ATPase and bc_1 complexes.

blue: small LH2 rings (blue)

blue/red: Z-shaped LH1/RC dimers form a linear array around the "equator" of the vesicle, determining the vesicle's diameter by their intrinsic curvature.

Geyer & Helms, Biophys J. (2006)



Summary

Integrated model of binding + photophysical + redox processes inside of chromatophore vesicles

Various experimental data fit well together

Equilibrium state.

How to model non-equilibrium processes?



Biophysical Journal Volume 99 July 2010 67-75

Photosynthetic Vesicle Architecture and Constraints on Efficient Energy Harvesting

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Photosynthesis: textbook view





Viewing the photosynthetic apparatus as a conversion chain



Thick arrows : path through which the photon energy is converted into chemical energy stored in ATP via the intermediate stages (rounded rectangles).

Each conversion step takes place in parallely working proteins. Their number N times the conversion rate of a single protein R determines the total throughput of this step.

- $\boldsymbol{\gamma}$: incoming photons collected in the LHCs
- E : excitons in the LHCs and in the RC
- e^-H^+ electron-proton pairs stored on the quinols
- $e^{\scriptscriptstyle -}$ for the electrons on the cytochrome c_2
- pH : transmembrane proton gradient
- H^+ : protons outside of the vesicle (broken outine of the respective reservoir).

Stochastic dynamics simulations: Molecules & Pools model



Round edges: **pools** for metabolite molecules

Rectangles: protein machines are modeled explicitly as multiple copies

fixed set of parameters

integrate rate equations with stochastic algorithm



Stochastic simulations of cellular signalling

Traditional computational approach to chemical/biochemical kinetics:

(a) start with a set of coupled **ODEs** (reaction rate equations) that describe the time-dependent concentration of chemical species,

(b) use some **integrator** to calculate the concentrations as a function of time given the rate constants and a set of initial concentrations.

Successful **applications** : studies of yeast cell cycle, metabolic engineering, whole-cell scale models of metabolic pathways (E-cell), ...

Major problem: cellular processes occur in very small volumes and frequently involve very small number of molecules.

E.g. in gene expression processes a few TF molecules may interact with a single gene regulatory region.

E.coli cells contain on average only 10 molecules of Lac repressor.



Include stochastic effects

(Consequence1) \rightarrow modeling of reactions as continuous fluxes of matter is no longer correct.

(Consequence2) Significant stochastic fluctuations occur.

To study the stochastic effects in biochemical reactions, stochastic formulations of chemical kinetics and Monte Carlo computer simulations have been used.

Daniel Gillespie (J Comput Phys 22, 403 (1976); J Chem Phys 81, 2340 (1977)) introduced the exact **Dynamic Monte Carlo (DMC)** method that connects the traditional chemical kinetics and stochastic approaches.



Basic outline of the direct method of Gillespie

(Step i) generate a list of the components/species and define the initial distribution at time t = 0.

(Step ii) generate a list of possible events E_i (chemical reactions as well as physical processes).

(Step iii) using the current component/species distribution, prepare a probability table $P(E_i)$ of all the events that can take place.

Compute the total probability

$$P_{tot} = \sum P(E_i)$$

 $P(E_i)$: probability of event E_i .

(Step iv) Pick two random numbers r_1 and $r_2 \in [0...1]$ to decide which event E_{μ} will occur next and the amount of time τ after which E_{μ} will occur.



Resat et al., J.Phys.Chem. B 105, 11026 (2001)

Basic outline of the direct method of Gillespie

Using the random number r_1 and the probability table,

the event E_{μ} is determined by finding the event that satisfies the relation

$$\sum_{i=1}^{\mu-1} P(E_i) < r_1 P_{tot} \leq \sum_{i=1}^{\mu} P(E_i)$$

The second random number r_2 is used to obtain the amount of time τ between the reactions

$$\tau = -\frac{1}{P_{tot}} \ln(r_2)$$

As the total probability of the events changes in time, the time step between occurring steps varies.

Steps (iii) and (iv) are repeated at each step of the simulation.

The necessary number of runs depends on the inherent noise of the system and on the desired statistical accuracy.



Resat et al., J.Phys.Chem. B 105, 11026 (2001)

reactions included in stochastic model of chromatophore





Stochastic simulations of a complete vesicle

Model vesicle: 12 LH1/RC-monomers

1-6 *bc*¹ complexes 1 ATPase

120 quinones 20 cytochrome c_2

integrate rate equations with:

- Gillespie algorithm (associations)
- Timer algorithm (reactions); 1 random number determines when reaction occurs

simulating 1 minute real time requires 1.5 minute on one opteron 2.4 GHz proc



simulate increase of light intensity (sunrise)

during 1 minute, light intensity is slowly increased from 0 to 10 W/m² (quasi steady state)

- \rightarrow there are two regimes
- one limited by available light
- one limited by bc_1 throughput





oxidation state of cytochrome c₂ pool





oxidation state of cytochrome c₂ pool





total number of produced ATP



high light intensity: interruptions are buffered up to 0.3 s duration



c₂ pool acts as buffer



At high light intensity, c2 pool is mainly oxidized.

If light is turned off, bc1 can continue to work (load c2s, pump protons, let ATPase produce ATP) until c2 pool is fully reduced.



What if parameters are/were unknown?

Bridging the Gap: Linking Molecular Simulations and Systemic Descriptions of Cellular Compartments

Tihamér Geyer*, Xavier Mol, Sarah Blaß, Volkhard Helms

Center for Bioinformatics, Saarland University, Saarbrücken, Germany

PLoS ONE (2010)

choose 25 out of 45 system parameters for optimization.

take 7 different non-equilibrium time-resolved experiments from Dieter Oesterhelt lab (MPI Martinsried).



Biochemistry 1995, 34, 15235-15247

Role of PufX Protein in Photosynthetic Growth of *Rhodobacter sphaeroides*.
1. PufX Is Required for Efficient Light-Driven Electron Transfer and Photophosphorylation under Anaerobic Conditions[†]



Parameters not optimized

Parameter	Value	Description	
bc1::kar(H+an)	10 ¹⁰ nm ³ s ⁻¹	rate for proton uptake from the cytoplasm by bc_1	
bc1∷ktr(e:Q₀=>FeS)	2.3 * 10 ³ s ⁻¹	rate for electron transfer from Q₀ to FeS	
$bc_1::k_{tr}(e:c_1 \Rightarrow c_2)$	10 ⁵ s ⁻¹	electron transfer rate from c_1 to bound cytochrome c_2	
$bc_1::k_{tr}(e:Q_0 \Longrightarrow b_L)$	104 s-1	electron transfer from Q_0 to b_L heme	
$bc_1::k_{tr}(e:b_L=>b_H)$	10 ⁴ s ⁻¹	electron transfer from $b_{\rm L}$ to $b_{\rm H}$ heme	
ΔΦ:: <i>V</i>	2.65 * 10 ⁴ nm ³	inner volume of the vesicle	
∆ Φ∷ A	5.28 * 10 ³ nm ²	membrane area (Q pool "volume")	
$\Delta \Phi$:: C_{Hin}	1.0 e	effective charge of a free proton in the vesicle	
$\Delta \Phi$:: C_{Hm}	1.0 e	effective charge of a proton on the titratable groups	
$\triangle \Phi$:: $C_{\text{prot.}}$	-1.0 e	effective charge of an e− translocated through an RC	
$\Delta \Phi$:: C_{ared}	-0.5 e	effective charge of a reduced cytochrome c_2	
$\Delta \Phi$:: C_{cox}	0.5 e	effective charge of an oxidized cytochrome c_2	
PR::Np	80	number of titratable groups in the vesicle	
PR::pK	5.0	pK of the titratable groups	
Ncare	10	number of dimeric core complexes (2 RC + 1 LHC)	
Mc1	10	number of cytochrome bc1 complexes	
NATPase	1	number of ATPases	
N _{c2}	20	total number of cytochrome c_2	
NQ	200	total number of quinones	

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Table S1: Model Parameters Not Included in the Optimization Process

Parameter optimization through evolutionary algorithm





Figure S1: Determining the Number of bc1 Complexes: Evolution of the Master Score

= 8

= 11

20

25 optimization parameters

Analyze 1000 best parameter sets among 32.800 simulations:

$$\langle P \rangle = \exp[\langle \log P \rangle]$$

 $\sigma^2 = \langle (\log P - \langle \log P \rangle)^2 \rangle$
 $P_{\min} = \exp[\langle \log P \rangle - \sigma]$
 $P_{\max} = \exp[\langle \log P \rangle + \sigma]$



parameter	units	< <i>P</i> >	P _{min} P _{max}	P _{min} / P _{max}
LHC::o	m ² W ¹ s ¹	6.22	6.026.42	0.94
LHC::No	1	1.31	0.81 2.13	0.38
LHC::k _D (E)	s ¹	1.9 * 10 ³	(1.13.8) * 10 ³	0.29
RC::kon(E)	s ¹	2.4 * 10 ⁶	(1.24.5) * 10 ⁶	0.27
RC::k₀n(H⁺)	nm³ s 1	1.4 * 10 ⁸	(1.31.6) * 10 ⁸	0.81
$RC:k_{on}(Q)$	nm² s 1	$6.0 * 10^4$	$(4.48.1) * 10^4$	0.54
RC::k _{off} (QH2)	s ¹	87	70108	0.65
RC::kon(c2red)	nm ³ s ¹	9.2 * 10 ⁵	(7.311.5) * 10 ⁵	0.63
RC::k _{off} (c2ox)	s ¹	2.2 * 10 ³	(1.63.0) * 10 ³	0.53
bc1::kon(QH2@Qo)	nm² s 1	1.2 * 10 ⁴	(0.791.7) * 10 ⁴	0.46
bc1::koff(Q@Qo)	s ¹	28.3	26.330.4	0.86
$bc_1::k_{ti}(\mathbf{Q};\mathbf{Q}_o - >\mathbf{Q}_i)$	s ¹	4.9 * 10 ³	$(3.66.7) * 10^3$	0.54
bc1::kon(Q@Qi)	mm ² s ⁻¹	6.7 * 10 ⁵	(4.510) * 10 ⁵	0.45
bc1::koff(QH2@Qi)	s ¹	86	68110	0.62
$bc_1::k_{ti}(QH2:Q_i = >Q_o)$	s ¹	3.8 * 10 ³	(2.65.5) * 10 ³	0.47
bc1::kon(c2ox)	nm ³ s ⁻¹	94 * 10 ⁶	(6.314) * 10 ⁶	0.47
bc1::koff(c2red)	s ¹	6.0 * 10 ³	(3.311) * 10 ³	0.30
bc1::koff(H+@Qo)	s ¹	2.4 * 10 ⁴	(1.34.3) * 10 ⁴	0.30
<i>bc</i> 1:: <i>k</i> 1(FeS:b = >c)	s ¹	3.9 * 10 ³	(3.15.1) * 10 ³	0.61
<i>bc</i> 1:: <i>k</i> 1(FeS:c=>b)	s ¹	2.8 * 10 ³	$(2.23.6) * 10^3$	0.61
$bc_1::k_{ti}(e:b_H = >Q_i)$	s ¹	7.7 * 10 ³	(5.012) * 10 ³	0.42
bc_1 :: Φ_0	mV	102	83114	0.73
ΔΦ::Uo	mV/e	10.3	9.511	0.85
$\Delta \Phi$:: $\Delta \Phi_0$	mV/pH	10	7.613.7	0.55
PR::pK	1	4.84	3.95.9	0.66

Sensitivity of master score

Decay rate of excitons in LHC



Absorption cross section light harvesting complex

Kinetic rate for hinge motion of FeS domain in bc1 complex

Some parameters are very sensitive, others not.



Three best-scored parameter sets

Score of individual parameter set *i* for matching one experiment:

$$s_i = \frac{C_i}{\sum \left(x(t_i) - f(t_i)\right)^2}$$

 $x(t_i)$: simulation result $f(t_i)$: smooth fit of exp. data

Master score for one parameter set: defined as product of the individual scores s_i





Different experiments yield different sensitivity



"importance score": Sum of the sensitivities P_{min} / P_{max} of all relevant parameters

Table 2. Importance scores and correlation coefficients between the master score and the respective individual scores of the experimental scenarios denoting the relative importance of each of the experiments for the parameter value optimization.

experiment	A7 cytc	A7 ΔΦ	A8 🛆 Ф	A9 cytc	B1 Q	86 P	B6 cytc	BC1
importance score	4.4	7.7	5.8	9.7	3.8	52	8.9	55
correlation	0.09	0.44	022	038	0.83	0.17	0.31	0.41

The importance scores are determined as the sums of the sensitivities of all relevant parameters against the individual scores (see table 52 for all the individual values). The correlation coefficients are obtained from a linear fit of the master score against the respective individual score.

Analysis could suggest new experiments that would be most informative!



Summary

Only 1/3 of the kinetic parameters previously known.

Stochastic parameter optimization converges robustly into the same parameter basin as known from experiment.

Two large-scale runs (15 + 17 parameters) yielded practically the same results.

If implemented as grid search, less than 2 points per dimension.

It appears enough to know 1/3 - 1/2 of kinetic rates about a system to be able to describe it quantitatively (IF connectivities are known).

