Mathematics of Cellular Network

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Exercise Sheet 3

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Send a single PDF attachment with your name(s), your solution(s). Send all source code via mail to nazarieh@ mpi-inf.mpg.de.

1 SIS model vs SIR model(25 points)

(a) Compare the SIR model with the SIS model and discuss about how they can be integrated into a more elaborate combined model.

(b) Consider the disease flu which is a common infectious disease. Suggest a way that SIR model can be extended to make it more realistic or more appropriate for this disease.

2 Estimate the parameters of an epidemic model(50 points)

The following table contains data about an epidemic outbreak. The community has 400 individuals, and the following table lists the number of susceptible, infected and recovered people during 15 days such that at each time $S + X + R = N$.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
s(t)	384	375	364	351	336	319	300	279	256	231	204	175	144	50	60
x(t)	12	20	30	42	56	72	90	110	132	156	182	210	240	200	120
r(t)	4	5	6	7	8	9	10	11	12	13	14	15	16	150	220

(a) Write a program to model the data with the SIR epidemic model. Use the equation $\frac{1}{x(t)}\frac{dx}{dt} = \beta s(t) - \gamma$ to plot $\frac{1}{x(t)}\frac{dx}{dt}$ versus s(t). The β and γ values are the slope and intercept of the plot respectively. Consider that in the SIR epidemic model we have:

- $\frac{ds}{dt} = -\beta sx$
- $\frac{dx}{dt} = \beta sx - \gamma x$
- $\frac{dr}{dt} = \gamma x$
- one can approximate $\frac{dx}{dt}$ using the formula $\frac{d(x)}{dt} \cong \frac{X(t+D)-X(t-D)}{2D}$ where D means day.

(b) Remove the two points(outliers) that deviate most from the general trend and recalculate the parameters.

3 Time-dependent properties of the SI model(25 points)

Consider an SI-type epidemic spreading on the giant component of a k-regular random graph, i.e., a configuration model network in which all vertices have the same degree k. Assume that some number c of vertices, chosen at random, are infected at time t = 0.

(a) Show using the results of section **Time-dependent properties of the SI model** that the probability of infections of every vertex increases at short times as $e^{\beta kt}$. (consider that $x(t) \sim e^{\beta k_1 t} v_1$, where k_1 and v_1 are the largest eigenvalue and eigenvalue of the adjacency matrix.)

(b) Show that within the first-order moment closure approximation of equation $\frac{dx_i}{dt} = \beta s_i \sum_j A_{ij} x_j = \beta(1 - x_i) \sum_j A_{ij} x_j$, the average probability of infection x of every vertex is the same and give the differential equation it satisfies.

(c) Hence show that $x(t) = \frac{ce^{\beta kt}}{n-c+ce^{\beta kt}}$.

(d) Find the time at which the **infection point** of the epidemics occurs, the point at which the rate of appearance of new disease cases stops increasing and starts decreasing.

Good Luck